Truthmaker Semantics

Chapter for the Blackwell Philosophy of Language Handbook

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My aim in the present chapter is to explain the basic framework of truthmaker or 'exact' semantics, an approach to semantics that has recently received a growing amount of interest, and then to discuss a number of different applications within philosophy and linguistics.

Here is a list of contents:

I. Theory

§1. Truthmakers in Metaphysics and Semantics

The idea of truthmaking is the idea of something on the side of the world - a fact, perhaps, or a state of affairs - verifying, or making true, something on the side of language or thought - a statement, perhaps, or a proposition. The idea of truthmaking has figured prominently in contemporary metaphysics and semantics. In its application to metaphysics, the thought has been that we can arrive at a satisfactory metaphysical view by attempting to ascertain *what it is*, on the side of the world, that renders true what we take to be true (as in Armstrong [1997], [2004]); and, on the semantical side, the thought has been that we can attain a satisfactory semantics for a given language by attempting to ascertain *how it is* that the sentences of the language are made true by what is in the world. In the former case, truthmaking serves as a conduit taking us from language or thought to an understanding of the world; and in the latter case, it has served as a conduit taking us from the world to an understanding of language.

I, along with other philosophers, have argued against truthmaking as a guide to metaphysics (Schnieder [2006], Horwich [2008], Fine [2012], §3). I have sometimes joked that truthmaking is fine as a guide to metaphysics as long as we junk the relata on the left, the things whose existence *makes* true, the relata on the right, the things *made* true, and the relation of *making* true. But my concern here is with truthmaking as a tool of semantics; and it is worth remarking, in this regard, that the task of discerning truthmakers may be helpful for the one project even if not at all helpful for the other.

Indeed, the general focus of the two projects is very different. If our aim is to understand the world, then our focus should be on the ultimate truthmakers, on *what* in the world ultimately makes something true, and the question of how the truthmakers make the statements of our language true is of no great concern. But if our aim is to understand language, then our focus should be on the immediate truthmakers, not the ultimate truthmakers, and the question of *how* they make the statements of the language true will be of greatest concern. Take the statement 'there is a chair over there', by way of example. For the metaphysical project, we may wish to give an account of the truthmakers for the statement in terms of elementary particles, let us say, and the real question of concern will be whether we can achieve a 'reduction' of the macroscopic to the microscopic. But for the semantical project, we can rest content with specifying the truthmakers in terms of ordinary macroscopic objects; and our concern in this case will not be with a reduction of the macroscopic to the microscopic but with what it is about the representational features of the statement itself that enables it to have the superficial truthmakers that it does. We see ripples on the surface of a pond; and our concern may be with what it is beneath the surface that causes the ripples or with how it is that the ripples play out over the surface, without regard for their cause.

§2. Truthmaker and Truth-conditional Semantics

There is a long tradition within philosophy, perhaps going all the way back to Frege [1892], of identifying the meaning of a statement with its truth-conditions, i.e. with the conditions under which it is true. However, a truth-conditional account of meaning can take various different forms and it may be worthwhile to locate truthmaker semantics within a general account of this sort.

One major line of division concerns the form of the truth-conditional claims themselves. On the clausal approach, especially associated with Davidson [1967], the truth-conditions of a statement are not entities as such but the clauses by which a theory of truth specifies when a statement is true. Thus a typical clause within such a theory might state that a conjunction '$A \wedge B$' is true when both of its conjuncts A and B are true. On the objectual approach, by contrast, the truth-conditions are objects, rather than clauses, which stand in a relation of truth-making to the statements they make true. Within such an account, therefore, there is both an ontology of truthmakers and a relation of truthmaking.

Within the objectual approach, a second major line of division concerns the nature of the truthmakers. Under the most familiar version of the objectual approach, the truth-conditions of a statement are taken to be possible worlds and the content of a statement may, accordingly, be identified with the set of possible worlds in which it is true. This gives rise to 'possible worlds semantics', which received its first systematic application to natural language in the work of Montague [1970].

Under a somewhat less familiar version of the objectual approach, the truth-conditions are not - or not, in general - taken to be possible worlds but states or situations - fact-like entities that serve to make up a world rather than being worlds themselves; and the content of a statement may, in this case, be identified with the set of verifying states or situations in which it is true. This gives rise to 'situation semantics', which received its first systematic development in the work of Barwise & Perry [1983].

The main difference between the two kinds of semantics turns on the question of completeness: the truth-value of any statement will be settled by a possible world (at least to the extent that it is capable of being settled), whereas the truth-value of a statement may not be settled by a state or situation. The state of the weather in New York, for example, will not settle whether it is raining in London.

Within situation semantics itself, there is a third major line of division. For whereas it is tolerably clear what it takes for a statement to be true at a possible world, considerable unclarity surrounds the question of what it is for a statement to be true in a situation. There are at least three different conceptions of the truthmaking relation that one might adopt. We might call them *exact*, *inexact* and *loose*; and they are successively broader. Thus each exact verifier of a statement is an inexact verifier and each inexact verifier a loose verifier.

Loose verification is a purely modal notion. A state or situation s will loosely verify a statement just in case the state necessitates the statement, i.e. just in case it is impossible that the state obtain and the statement not be true. Exact and inexact verification, by contrast, require that there be a relevant connection between state and statement. With inexact verification, the state should at least be *partially* relevant to the statement; and with exact verification, it should be *wholly* relevant. Thus the presence of rain will be an exact verifier for the statement 'it is rainy'; the presence of wind and rain will be an inexact verifier for the statement 'it is rainy', though not an exact verifier; and the presence of wind will be a loose verifier for the statement 'it is rainy or not rainy' (since the statement is true no matter what), while failing to be an inexact verifier. Loose and inexact verification are monotonic or 'hereditary': if a state necessitates or is partially relevant to the truth of a statement, then so is any more comprehensive state. But exact verification is not hereditary; the statement 'it is rainy' will be verified by the presence of rain, for example, by not by the presence of rain and wind.

Here, in summary form, is a diagram of the various options:

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              Truth-conditional Semantics
          /                                  \
     Clausal                              Objectual                    (Form)
                                        /           \
                                 Worldly             Stately           (Relata)
                                                  /     |     \
                                              Loose  Inexact  Exact    (Relation)
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And within this framework, what I am calling 'truthmaker semantics' lies to the far right, with verification objectual, stately and exact.

§3. Exact Verification

When we survey the literature on situation semantics, we find that there has been little interest in the loose version of the semantics (though the work of Humberstone [1981] and Rumfitt [2012] is something of an exception). Most of the work - beginning with Barwise & Perry [1983] and extending to more recent work in the formal semantics for natural language (as typified by Kratzer [2014]) - has adopted the inexact approach or variant of it.

The exact approach, on the other hand, has had a somewhat checkered career. Van Fraassen's 'Facts and Tautological Entailments' [1969] was the first paper to present an exact semantics for classical logic. But it has been the fate of the semantics presented in that paper to be constantly rediscovered. Thus one reincarnation has been in the work of L. K. Schubert and his collaborators (Hwang C.H. & Schubert L.K., [1993]); another in the work of Stelzner [1992] on deontic logic; and another in some of the work of the school of Inquisitive Semantics at Amsterdam (Groenendijk & Roelofsen [2010]). I myself hit upon the idea of the semantics in the late 1960's and briefly mentioned its application to the problem of 'Disjunctive Simplication' in my review of Lewis' book on counterfactuals (Fine [1975]). However, it is only very recently that the full range and potential of the approach - as evidenced by the work of Aloni & Ciardelli [2013], Angell ([1977], [1989], [2002]), Fine ([2012b,c], [2013], [2014a,b], [2015a,b,c,d,e,f]), Gemes ([94], [97]), Moltmann [2007], Van Rooij [2013] and Yablo [2014], among others - has begun to be appreciated.

It should also be mentioned, though this may not be evident to those working within only one of the respective fields, that there is an intimate connection between the recent semantical work on truthmaking and the recent metaphysical work on ground (Correia & Schnieder [2012] is a recent collection of papers on ground). For if some statements A_1, A_2, ... are to constitute a ground for another statement C, then they must be wholly relevant to the other statement's being the case; and, indeed, we might think of the notion of exact verification as being obtained through a process of ontological and semantic ascent from a claim of ground. For we first convert the statements A_1, A_2, ... into the corresponding facts f_1, f_2, ... (that A_1, A_2, ..., obtain) and then take the sum f of the facts f_1, f_2, ... to be an exact verifier for the truth of C. It is my belief that it is the notion of ground rather than the notion of truthmaking that is most relevant to the metaphysical project of discerning the nature of reality; and so it is of interest that this notion, of fundamental significance to metaphysics, can be 'reworked' in this way into a notion of fundamental significance to semantics.

§4. State Spaces

In setting up the possible worlds semantics, we suppose given a 'pluriverse' of possible worlds; and, in applying the semantics, there is no need to suppose that possible worlds have any internal mereological structure - they can simply taken to be undifferentiated 'blobs', perhaps externally related by certain accessibility relations but with no internal mereological structure of their own.

For the purposes of setting up the truthmaker semantics, we suppose given a 'state space' of states. But it will be important, in applying the semantics, to suppose that the states, in contrast to possible worlds, are endowed with mereological structure. We must allow, for example, that one state may be a part of another, as when the presence of rain is part of the presence of rain and the wind, or that two states may fuse to a single composite state, as when the presence of rain and the presence of wind fuse to the presence of rain and wind.

It is also important in applying the semantics to appreciate that the term ‘state’ is a mere term of art and need not be a state in any intuitive sense of the term. Thus facts or events or even ordinary individuals could, in principle, be taken to be states, as long as they are capable of being endowed with the relevant mereological structure and can be properly regarded as verifiers. We allow the states, whatever they might be, to be possible as well as actual (as with Gore wining the presidency in 2001; and we even allow impossible states (as with Gore both winning and losing the presidency in 2001). Thus verification will have a counterfactual flavor; a verifying state is one that *would* make a given statement true were to obtain, not necessarily one that *does* make the statement true.

From a purely mathematical point of view, we may take a *state space* to be an ordered pair $(S, \sqsubseteq)$, where S (states) is a non-empty set and $\sqsubseteq$ (part) is a binary relation on S. We suppose that the relation $\sqsubseteq$ is a *partial order*, i.e. that it conforms to the following three conditions for any states s, t and u of S:

(i) Reflexivity: $s \sqsubseteq s$
(ii) Anti-symmetry: $s \sqsubseteq t$ & $t \sqsubseteq s$ implies $s = t$
(iii) Transitivity: $s \sqsubseteq t$ & $t \sqsubseteq u$ implies $s \sqsubseteq t$.

We wish to impose a further condition on a state space, although this will call for two addiitonal definitions. Given a subset $T \subseteq S$ of states, we say that s is an *upper bound of T* if it contains each state of T, i.e. if $t \sqsubseteq s$ for each $t \in T$, and we say that s is a *least upper bound* (lub) *of T* if s is an upper bound of T and if it is included in any upper bound of T, i.e. if $s \sqsubseteq s'$ for any upper bound s' of T. We then require that a state space be *complete* in the sense that every subset $T \subseteq S$ of states have a least upper bound.

The least upper bound of $T \subseteq S$ is unique (since if s and s' are least upper bounds, then $s \sqsubseteq s'$ and $s' \sqsubseteq s$ and so, by anti-symmetry, $s = s'$). We denote it by $\bigsqcup T$ and call it the *fusion of T* (or of the members of T). When $T = \{t_1, t_2, ...\}$, we may write $\bigsqcup T$ more perspicuously in the form $t_1 \sqcup t_2 \sqcup ...$; and so, in particular, $s \sqcup t$ will be the least upper bound of s and t. For many applications, we need only assume the existence of $s \sqcup t$ and not the existence of $t_1 \sqcup t_2 \sqcup ...$ for arbitrary t_1, t_2, ...

The state space $(S, \sqsubseteq)$, as we have defined it, does not embody the distinction between possible and impossible states; for all that we have said, each state in S might be an impossible state. In order to give recognition to the distinction, we may take a *modalized state space* to be an ordered triple $(S, S^\diamond, \sqsubseteq)$, where $(S, \sqsubseteq)$ is a state space as before and $S^\diamond$ (possible states) is a non-empty subset of S. We make one assumption about $S^\diamond$, namely:

<u>Closure under Part</u> $t \in S^\diamond$ whenever $s \in S^\diamond$ and $t \sqsubseteq s$.

Parts of possible states are also possible states.

The interest of modalized state spaces is that they enjoy both a mereological structure (as represented by $\sqsubseteq$) and a modal structure (as represented by $S^\diamond$); and the interaction of the two enables us to define many notions of significance. We may say, in particular, that two states s and t are *compatible* if their fusion $s \sqcup t$ is a possible state (i.e. a member of $S^\diamond$) and *incompatible* if their fusion $s \sqcup t$ is not a possible state. Thus the possible state of my being cold and the possible state of my being hungry will presumably be compatible since the state of my being cold and hungry is a possible state, while the possible state of my being cold and the possible state of my being hot will presumably be incompatible since the state of my being both cold and hot is not a possible state.

We may also employ the resources of a modalized state space to say when a state corresponds to a possible world. For given a modalized space (S, $S^{\Diamond}$, $\sqsubseteq$), we may say that the state s is a *world-state* if it is possible and if any state is either a part of s or incompatible with s. Thus a world state must positively include or exclude any other state. Given the notion of a world-state, we may then say that the space is (S, $S^{\Diamond}$, $\sqsubseteq$) is a *W-space* if every possible state of S is part of a world-state. Thus a W-space will, in effect, contain the pluriverse of possible worlds. However, very few applications require the assumption that the state space be a W-space and so, from this perspective, the postulation of possible worlds is a gratuitous assumption that serves no real purpose.

It should be noted that our approach to states is highly general and abstract. We have formed no particular conception of what they are; and nor have we assumed that there are 'atomic' states, from which all other states can be obtained by fusion. Nearly all of the existing literature on the topic has failed to adopt such a neutral perspective. Thus states are often identified with sets of possible worlds (where the worlds themselves might be identified with sets of sentences) or it is assumed that all states are constructed from atomic states which are somehow isomorphic with the atomic sentences of the language under consideration.

Nothing is gained by this lack of generality or abstraction and a great deal is lost. For one thing, the particular identifications or assumptions may not be appropriate in certain contexts. One might well think, for example, that a progressive statement such as 'this is moving' is not made true by any atomic state but by the motions of the object over successively shorter intervals of time; and if one takes a state to be a set of possible worlds, then one denies oneself the possibility of distinguishing between different necessary states or different impossible states. The technical development of the subject also requires a more abstract approach. For one will want to perform certain constructions on state spaces (forming product spaces, for example, or congruent spaces) in which the special identifications of or restrictions on the original spaces are lost.

The abstract approach to modal logic championed by Kripke's early work (in which possible worlds are simply regarded worlds as arbitrary points, rather than as models or sets of sentences) has been a great boon to the formal development of modal logic; and it is to be hoped that future researchers will appreciate that there are similar benefits to be gained by adopting a more abstract approach to the truthmaker framework as well.

§5. Sentential Semantics for Exact Verification

A good test for any proposed semantical framework is its ability to deal with classical sentential logic; and so let me show how we might give such a semantics within the truthmaker framework.

Recall the standard possible worlds semantics for sentential logic. For the case of negative, conjunctive and disjunctive statements we have the following clauses:

(ii) a world verifies $\neg$A iff it does not verify A;

(iii) a world verifiers a conjunction A $\wedge$ B iff it verifies A and verifies B;

(iv) a world verifies a disjunction A $\vee$ B iff it verifies A or verifies B.

Let us now give the corresponding clauses under the truthmaker approach. Our aim is not simply to say when a statement is true *at* a world but to say what it is *in* the world that makes it true and in such a way that the truthmaker is wholly relevant to the statement it makes true. To this end, we shall find it helpful to give separate clauses for when a statement is verified and for when it is falsified.[1] Here are the clauses for negation, conjunction and disjunction, which are split into two parts - one for verification and the other for falsification:

(ii)′ A state verifies a negative statement ¬A just in case it falsifies the negated statement A; and

a state falsifies the negative statement ¬A just in case it verifies the negated statement A.

(iii)′ A state verifies a conjunction A $\wedge$ B just in case it is the fusion of states that verify the respective conjuncts A and B; and

a state falsifies the conjunction A & B just in case it falsifies A or falsifies B.

(iv)′ A state verifies a disjunction A $\vee$ B just in case it verifies one of the disjuncts A or B; and

a state falsifies the disjunction A $\vee$ B just in case it is the fusion of states that falsify the respective disjuncts A and B.

Clearly, these clauses are all very plausible, once we have in mind that our interest is in the notion of *exact* verification: what exactly verifies a conjunction is the fusion of the verifiers for its conjuncts; what exactly verifies a disjunction is a verifier for one of the disjuncts; and similarly for the falsification of a conjunction or of a disjunction.

Let us now provide a more technical exposition of the semantics. We suppose given an infinitude of atomic sentences $p_1, p_2, \ldots$. Formulas of the sentential language are then constructed from the atomic sentences using the Boolean operators $\wedge$, $\vee$, and $\neg$ in the usual way.

A (*state*) *model* $\boldsymbol{M}$ is an ordered triple $(S, \sqsubseteq, |\cdot|)$, where $(S, \sqsubseteq)$ is a state space, as before, and $|\bullet|$ (valuation) is a function taking each sentence letter p into a pair (V, F) of subsets of S - intuitively, the set V of its verifiers and the set F of its falsifiers.

When a model $\boldsymbol{M} = (S, S^{\Diamond}, \sqsubseteq, |\bullet|)$ is constructed over a modalized state space $\boldsymbol{S} = (S, S^{\Diamond}, \sqsubseteq)$, one might want to take into account the interaction between the verifiers V and the falsifiers F of any given atomic sentence p. There are then two plausible conditions that might be imposed:

<u>Exclusivity</u> No verifier is compatible with a falsifier (i.e. no member of V is compatible with a member of F);

<u>Exhaustivity</u> Any possible state is compatible with a verifier or with a falsifier (i.e., each possible state is compatible with a member of V or with a member of F).

Exclusivity corresponds to the assumption that no statement is both true and false (which is how things would be if a verifier were compatible with a falsifier); and Exhaustivity corresponds to the assumption that every statement is either true or false (since no possible state could exclude the statement being either true or false). Of course, either of these assumptions could be

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[1] One might also provide a truthmaker semantics for *intuitionistic* sentential logic (Fine [2014a]) and, in this case, it is not necessary to state separate clauses for the verification and falsification of an arbitrary statement.

dropped if one wanted to allow either truth-value gluts (a statement being both true and false) or truth-value gaps (a statement being neither true nor false).

Given a model $\boldsymbol{M} = (S, \sqsubseteq, |\bullet|)$, we may now define what it is for an arbitrary formula A to be *verified by* a state s (s ||- A) or *falsified by* a state s (s -|| A):

(i)$^+$ s ||- p if $s \in |p|^+$;
(i)$^-$ s -|| p if $s \in |p|^-$;
(ii)$^+$ s ||- $\neg$B if s -|| B;
(ii)$^-$ s -|| $\neg$B if s ||- B;
(iii)$^+$ s ||- B $\wedge$ C if for some states t and u, t ||- B, u ||- C and $s = t \sqcup u$;
(iii)$^-$ s -|| B $\wedge$ C if s -|| B or s -|| C;
(iv)$^+$ s ||- B $\vee$ C if s |= B or s |= C;
(iv)$^-$ s -|| B $\vee$ C if for some t and u, t -|| B, u -|| C and $s = t \sqcup u$.

Here (ii)$^+$ and (ii)$^-$ correspond to (ii)′ above, (iii)$^+$ and (iii)$^-$ to (ii)′, and (iv)$^+$ and (iv)$^-$ to (iv)′.

§6. Some Features and Consequences of the Semantics

I should like to discuss some noteworthy features of the semantics, especially in so far as they stand in contrast to other, more familiar, semantical schemes.

One immediate odd consequence of the clauses is that p $\wedge$ p may not have the same verifiers as p. For suppose that s and t are the two verifiers of p, where neither is a part of the other. Then $s \sqcup t$ is a verifier for p $\wedge$ p by clause (iii)$^+$ but not a verifier for p. I believe that this odd consequence is not a problem for natural language applications but that it may be a problem for more theoretical applications. If, for example, our interest is in the partial truth or confirmation or verisimilitude of a statement, then it seems to me that we would not want to distinguish in this way between the statements p and p $\wedge$ p.

We may avoid this odd consequence, if we wish, by allowing that a verifier for A $\wedge$ B should also be a verifier for A $\vee$ B (and, likewise, by allowing that a falsifier of A $\vee$ B should also be a falsifier for A $\wedge$ B). Thus (iii)$^-$ and (iv)$^+$ now become:

(iii)*$^-$ s -|| B $\wedge$ C if s -|| B or s -|| C or s -|| B $\vee$ C
(iv)*$^+$ s ||- B $\vee$ C if s |= B or s |= C or s ||- B $\wedge$ C.

and we obtain what might be called the 'inclusive' semantics. It is characteristic of the inclusive semantics that the set $\{s \in S$: s ||- A$\}$ of verifiers of any statement A should always be *closed under fusion*, i.e. if $s_1, s_2, \ldots$ are one or more verifiers of A then so is their fusion $s_1 \sqcup s_2 \sqcup \ldots$.

There are a number of other variants of the clauses that might also be considered (as in Fine [2015c] or Groenendijk & Roelofsen [2010]) but, in the interests of simplicity, we shall usually confine our attention in what follows to the original non-inclusive clauses.

It is evident from the above account that exact verification need not be hereditary. Indeed, there is nothing to prevent an atomic sentence p being exactly verified by a single state p and yet not verified by any state $p' \sqsupseteq p$. However, if the verifiers of all the atomic sentences p_1, p_2, ... are hereditary, i.e. if s ||- p_k and $s' \sqsupseteq s$ implies s' ||- p_k, then it can be show that the verifiers of all formulas A whatever will be hereditary, i.e. that s ||- A and $s' \sqsupseteq s$ implies s' ||- A. Thus the failure of persistence arises from the behavior of the atomic sentences and is not attributable to the behavior of the sentential connectives.

It should also be evident that there is no reason in general to think that the exact verifiers will be minimal. Say that the state s *minimally verifies* the formula A if s exactly verifies A and if no proper part of s exactly verifies A (i.e. if $s' \sqsubseteq s$ and $s' \Vdash$ A implies $s' = s$). Now suppose that p is the sole verifier of p and q the sole verifier of q, with $q \not\sqsubseteq p$. Then p and $p \sqcup q$ are both verifiers of p $\vee$ (p $\wedge$ q), with $p \sqcup q$ a non-minimal since it contains the verifier p as a proper part. Indeed, there is no reason to suppose that a statement with verifiers need have any minimal verifiers at all. In the case of 'this is moving', for example, we may well maintain that any verifier (the motion of the object through an interval of time) will contain another verifier as a proper part.

There has been a persistent tendency in the literature (we might call it 'minimalitis') to start off with a hereditary notion of verification and then attempt to get the corresponding notion of minimal verification, or some variant of it, to do the work of exact verification (as in the account of 'exemplification' in Kratzer [2014]). But if I am correct, all such attempts are doomed to failure. The relevant sense in which an exact verifier is wholly relevant to the statement it makes true is not one which requires that no part of the verifier be redundant but is one in which each part of the verifier can be seen to play an active role in verifying the statement. Thus the verifier $p \sqcup q$ of p $\vee$ (p $\wedge$ q) can be seen to play such an active role, even though the part q is redundant, because of its connection with the second disjunct (p $\wedge$ q).

It is important to note that within the present semantics (and this is also true of a number of variants), two formulas A and B may have the same verifiers while ¬A and ¬B do not have the same verifiers. For let A be the formula p $\wedge$ (q $\vee$ r) and B the formula (p $\wedge$ q) $\vee$ (p $\wedge$ r). Then it is readily verified that A and B will have the same verifiers (in any model). For the verifiers of p $\wedge$ (q $\vee$ r) will be the fusions $p \sqcup s$ of a verifier p for p and a verifier s for (q $\vee$ r) by clause (iii)$^+$ above and hence will be the fusions $p \sqcup q$ of a verifier p for p and a verifier q for q or the fusions $p \sqcup r$ of a verifier p for p and a verifier r for r by clause (iv)$^+$, and these are exactly the verifiers of (p $\wedge$ q) $\vee$ (p $\wedge$ r) (again by (iii)$^+$ and (iv)$^+$). However, the verifiers for ¬A and ¬B may not be the same in this case. For suppose $\bar{p}$ is the sole falsifier of p, $\bar{q}$ of q and $\bar{r}$ of r. Then by clauses (iii)$^-$ and (iv)$^-$, $\bar{p} \sqcup \bar{r}$ is a falsifier of (p $\wedge$ q) $\vee$ (p $\wedge$ r) but not of p $\wedge$ (q $\vee$ r) (unless, of course, $\bar{p} \sqcup \bar{r}$ happens to be identical to $\bar{p}$ or to $\bar{q} \sqcup \bar{r}$).

This means that it is essential to define verification by means of a double induction on verification and falsification, as we did above, since the verifiers of the negation statement ¬A cannot in general be determined from the verifiers of the negated statement A. It also means that we must complicate our definition of *proposition*. Within the possible worlds framework, the proposition (or content) expressed by a bivalent statement A can be identified with the set of worlds at which it is true. There is no need to bring in the worlds at which it is false, since these will simply be the worlds at which it is not true. Within the present framework, we might similarly identify the proposition expressed by a bivalent statement A with the set of its verifiers $V = \{s \in S: s \Vdash \text{A}\}$. We thereby obtain what I call a *unilateral* conception of propositionhood. But this conception will not be adequate if we wish to able to discern the proposition expressed by ¬A from the proposition expressed by A. In this case, we should adopt a *bilateral* conception of propositionhood, according to which the proposition expressed by a statement A is a *pair* (V, F) of sets of states consisting of its set of verifiers $V = \{s \in S: s \Vdash \text{A}\}$) and its set of falsifiers $F = \{s \in S: s \dashv\mid \text{A}\}$ (in effect, this was already presupposed in our previous account of the valuation function |•|). Under the unilateral conception of propositions, we might then take the conjunction

$P \wedge Q$ of two propositions P and Q to be $\{p \sqcup q: p \in P$ and $q \in Q\}$ and we might take their disjunction $P \vee Q$ to be $P \cup Q$ (i.e. $\{s: s \in P$ or $s \in Q\}$; and, under the bilateral conception of propositions, we might take the negation of the proposition (P, P') to be (P', P), the conjunction of the propositions (P, P') and (Q, Q') to be $(P \wedge Q, P' \vee Q')$, and their disjunction to be $(P \vee Q, P' \wedge Q')$ (in conformity with clauses (ii)$^+$ - (iv)$^-$ above).

One interesting aspect of the present approach is that it enables us to *define* the notions of inexact and loose verification. Thus we may say that *s inexactly verifies* A - in symbols, s ||> A - if s contains an exact verifier of A, i.e. if for some state $s' \sqsubseteq s$, s' ||- A, and we may say (within a modalized model) that *s loosely verifies* A - in symbols, s |= A - if s is incompatible with any exact falsifier of A, i.e. if s is incompatible with t whenever t -|| A. It turns out that the 'reverse' definitions are not possible: exact verification cannot be defined in terms of inexact verification; and inexact verification cannot be defined in terms of loose verifications. Thus the different notions of verification - loose, inexact and exact - involve a progressively greater commitment to what might be involved in the verification of a given statement; and we obtain the greatest flexibility in developing a theory of verification by taking the exact notion as primitive and seeing the other notions as off-shoots of the exact notion.

A further interesting aspect of the approach is that it naturally gives rise to two notions of consequence. There are two natural ways of defining the relation of consequence between propositions P and Q - in terms either of conjunctive part or of disjunctive part. We may say that P is a *conjunctive part of* Q if Q is the proposition $P \wedge R$ for some proposition R; and we may say that P is a *disjunctive part of* Q if Q is the proposition $P \vee R$ for some proposition R. We may then define Q to be a consequence of P either when Q is a conjunctive part of P or when P is a disjunctive part of Q.

Under the identification of classical truth-functionally equivalent propositions (and hence, in particular, under the possible worlds approach), the two relations of consequence will coincide. For if Q is a conjunctive part of P, i.e. if P is the proposition $Q \wedge R$ for some R, then Q will be the proposition $P \vee Q$ (given the classical truth-functional equivalence of Q to $(Q \wedge R) \vee Q$) and hence P will be a disjunctive part of Q; and if P is a disjunctive part of Q, i.e. if Q is the proposition $P \vee R$, then P will be the proposition $Q \wedge P$ (given the classical truth-functional equivalence of P to $(P \vee R) \wedge P$) and hence Q will be a conjunctive part of P.

However, within the truthmaker approach the two relations come apart. Let us, for simplicity, work with a unilateral conception of propositions, under which a proposition is identified with its set of verifiers. Then the relation of P's being a disjunctive part of Q will correspond most closely to the classical notion of consequence, since this is the relation that will hold between P and Q when every verifier of P is a verifier of Q (just as Q will be a consequence of P under the possible worlds approach when every world of P is a world of Q). But the notion of P's being a conjunctive part of Q will be very different and has no real counterpart within the possible worlds approach. P will be a conjunctive part of Q only if (and, under certain simplifying assumptions, if and only if) the following two conditions obtain:

(i) every verifier of P is included in a verifier of Q; and

(ii) every verifier of Q contains a verifier of P.

Thus the proposition $P = \{p, q\}$ will be a conjunctive part of the proposition $Q = \{p \sqcup r, p \sqcup s, q \sqcup r, q \sqcup s\}$, since Q is the conjunction of $P \wedge R$, for R the proposition $\{r, s\}$. But, of course, Q is not, in general, a disjunctive part of *P*.

The relation of conjunctive part corresponds to the intuitive notion of *partial content* - of what is conveyed, in whole or part, by what is said. Thus in saying that Fido is a cocker spaniel, I convey, in part, that he is a spaniel and that he is a dog, but I do not convey, even in part, that he is a dog or a cat. In the one case, the content of ‘Fido is a spaniel’ is part of the the content of ‘Fido is a cocker spaniel’ while, in the other case, the content of ‘Fido is a dog or a cat’ is not part of the content of ‘Fido is a dog’.

The existence of the two relations of consequence may be of some methodological significance to the study of linguistics. For it is often assumed that intuitions of validity provide a key piece key of data (some might think, *the* key piece of data) in the construction of a formal semantics for natural language. But, if I am right, then we should be somewhat more sensitive to the different inferential relationships that might be in play and it will be of particular importance to distinguish the subclass of inferential relationships that preserve content (as in the example above) and not merely truth.

There is one final aspect of truthmaker semantics which I should mention and which is of the greatest importance. It will have been noted that in specifying the verifiers of truth-functionally complex statements, we have not restricted ourselves to possible states. For given that p is a verifier for p and q is a verifier for q, we take $p \sqcup q$ to be a verifier for p $\wedge$ q even if p and q are incompatible and $p \sqcup q$ is therefore an impossible state. But it is not just that we do not restrict ourselves to possible states, we do not even make use of the distinction between possible and impossible states. The distinction is simply irrelevant in specifying the semantics for the connectives.

Of course, the distinction between possible and impossible will be required in providing an account of modal notions. We may want to say, for example, that ‘necessarily, A’ is true if every *possible* state is compatible with a verifier of A. Or again, we will say that *s* loosely verifiers A (a modal notion) if *s* is *incompatible* with any falsifier of A or that C is a classical consequence of A (another modal notion) if every loose verifier of A is a loose verifier of C. But the present point of view is that there is nothing in the general notion of content or meaning or in the most general logical devices that requires us to draw the distinction between possible and impossible states. This freedom from the modal thinking that has been so characteristic of the more usual approaches to semantics is, I believe, one of the most distinctive and liberating aspects of the present approach.

§7. Quantifiers

I have so far said nothing about the quantifiers. There are a number of different options for extending the clauses for the connectives to the quantifiers. But rather than considering them all, let me discuss one especially simple option, with an indication of how it might be extended to other cases.

One very general strategy for providing a semantics for quantificational statements is to reduce them to the corresponding truth-functional statements. Thus suppose the universal quantifier $\forall$x ranges over the individuals a_1, a_2, ... Then given that a_1, a_2, ... are constants for the corresponding individuals a_1, a_2, ... , we might take the content of $\forall x\varphi(x)$ to be the same as the content of the conjunction $\varphi(a_1) \wedge \varphi(a_2) \wedge ...$; and, similarly, we might take the content of the existential quantification $\exists x\varphi(x)$ to the same as that of the disjunction $\varphi(a_1) \vee \varphi(a_2) \vee \ldots$.

If we apply this general strategy to the present case, we arrive at the following clauses for the two quantifiers:

(v) a state verifiers $\forall x\varphi(x)$ if it is the fusion of verifiers of its instances $\varphi(a_1)$, $\varphi(a_2)$, ...;
a state falsifies $\forall x\varphi(x)$ if it falsifies one of its instances.

(vi) a state verifies $\exists x\varphi(x)$ if it verifies one of its instances $\varphi(a_1)$, $\varphi(a_2)$, ...;
a state falsifies $\exists x\varphi(x)$ if it is the fusion of falsifiers of its instances.

Within a more formal treatment, we might introduce variables, predicates and quantifiers into the language and define a formula (of the resulting first-order language) in the usual way.

A *model* $\boldsymbol{M}$ is now an ordered quadruple $(S, A, \sqsubseteq, |\bullet|)$, where $(S, \sqsubseteq)$ is a state space, as before, A (individuals) is a non-empty set, and $|\bullet|$ (valuation) is a function taking each n-place predicate F and any n individuals $a_1, a_2, \ldots, a_n$ of A into a pair (V, F) of subsets of S - where, intuitively, V is the set of states which verifies F of $a_1, a_2, \ldots, a_n$ and F is the set of states which falsify F of $a_1, a_2, \ldots, a_n$.

We may introduce constants $a_1, a_2, \ldots$ into the language, one for each of the distinct individuals $a_1, a_2, \ldots$ that compose A. We then have the following clauses for the closed atomic and quantificational formulas:

(i)$^+$ $s \Vdash Fa_1a_2 \ldots a_n$ if $s \in |F, a_1, a_2, \ldots, a_n|^+$;

(i)$^-$ $s \dashv\!\!\| Fa_1a_2 \ldots a_n$ if $s \in |F, a_1, a_2, \ldots, a_n|^-$

(v)$^+$ $s \Vdash \forall x\varphi(x)$ if there are states $s_1, s_2, \ldots$ with $s_1 \Vdash \varphi(a_1)$, $s_2 \Vdash \varphi(a_2)$, ... and $s = s_1 \sqcup s_2 \sqcup \ldots$;

(v)$^-$ $s \dashv\!\!\| \forall x\varphi(x)$ if $s \dashv\!\!\| \varphi(a)$ for some individual $a \in A$.

(vi)$^+$ $s \Vdash \exists x\varphi(x)$ if $s \Vdash \varphi(a)$ for some individual $a \in A$;

(vi)$^-$ $s \dashv\!\!\| \exists x\varphi(x)$ if there are states $s_1, s_2, \ldots$ with $s_1 \dashv\!\!\| \varphi(a_1)$, $s_2 \dashv\!\!\| \varphi(a_2)$, ... and $s = s_1 \sqcup s_2 \sqcup \ldots$.

Just as we can allow for a more inclusive clause for the falsification of a conjunction or the verification of a disjunction (clauses (iii)$^{*-}$ and (iv)$^{*+}$ from §6 above), so we can allow for a more inclusive clause for the falsification of a universal quantification and the verification of an existential quantification:

(v)$^{*-}$ $s \dashv\!\!\| \forall x\varphi(x)$ if for some distinct individuals $a_{k1}, a_{k2}, \ldots, s_{k1} \dashv\!\!\| \varphi(a_{k1}), s_{k2} \dashv\!\!\| \varphi(a_{k2}), \ldots,$ and $s = s_{k1} \sqcup s_{k1} \sqcup \ldots$;

(vi)$^{*+}$ $s \Vdash \exists x\varphi(x)$ if for some distinct individuals $a_{k1}, a_{k2}, \ldots, s_{k1} \Vdash \varphi(a_{k1}), s_{k2} \Vdash \varphi(a_{k2}), \ldots,$ and $s = s_{k1} \sqcup s_{k1} \sqcup \ldots$.

The difference between the two sets of clauses is that, in the first case, a universal quantification is only falsified and an existential quantification only verified via a single instance while, in the second case, the quantificational statements are verified or falsified via one or more instances.

One problem with these clauses is that they presuppose a fixed domain of individuals. For suppose that the actual individuals are $a_1, a_2, \ldots$ and that a is a merely possible individual (distinct from each of $a_1, a_2, \ldots$). Then in a possible world in which a exists, the truth of the instances $\varphi(a_1)$, $\varphi(a_2)$, ... is not sufficient to guarantee the truth of $\forall x\varphi(x)$ and hence the fusion of verifiers for $\varphi(a_1)$, $\varphi(a_2)$, ... need not be a verifier for $\forall x\varphi(x)$ (this is a familiar problem, going back to the early days of logical atomism).

There are a number of ways in which one might attempt to solve this problem within the present framework. One might drop the requirement that a verifying state should necessitate the statement that it verifies, for example, or one might treat the universal statement $\forall x\varphi(x)$ as equivalent, in effect, to $\Pi x(\neg Ex \vee \varphi(x))$, where Πx is an 'outer' quantifier ranging over a fixed domain of all actual and possible individuals and E is an existence predicate - thereby reducing the variable domain case to the fixed domain case. My own favored solution is to suppose that, for each subdomain B of possible individuals, there is a totality state τ_B to the effect that the individuals of B are exactly the individuals that there are. The reader should note that the totality state τ_B is taken to exist even when it is not possible for the individuals of B to be exactly the individuals that there are.[2] The two clauses for the universal quantifier then take the following form:

$s \Vdash \forall x\varphi(x)$ if there is a subset $B \subseteq A$ composed of the distinct individuals $b_1, b_2, ...$ and states $s_1, s_2, ...$ such that $s_1 \Vdash \varphi(b_1)$, $s_2 \Vdash \varphi(b_2)$, ... and $s = \tau_B \sqcup s_1 \sqcup s_2 \sqcup ...$;

$s \dashv\!| \forall x\varphi(x)$ if there is a subset $B \subseteq A$, an individual b from B and a state s' such that $s' \dashv\!| \varphi(b)$ and $s = \tau_B \sqcup s'$.

And similarly for the existential quantifier.

Another problem is that we have only provided clauses for unrelativized quantification. But one might also want to deal with relativized quantifiers - as in $\forall x[\varphi(x): \psi(x)]$ (all φ's ψ) and $\exists x[\varphi(x): \psi(x)]$ (some φ's ψ). To take care of this problem, one might relativize the totality condition to the suitably defined content $|\varphi(x)|$ of the restrictor $\varphi(x)$. The clauses for the universal quantifier would then take the following form:

$s \Vdash \forall x[\varphi(x): \psi(x)]$ if there is a subset $B \subseteq A$ composed of the distinct individuals $b_1, b_2, ...$ and states $s_1, s_2, ...$ such that $s_1 \Vdash \psi(b_1)$, $s_2 \Vdash \psi(b_2)$, ... and $s = \tau_{|\varphi(x)|,B} \sqcup s_1 \sqcup s_2 \sqcup ...$;

$s \dashv\!| \forall x[\varphi(x): \psi(x)]$ if there is a subset $B \subseteq A$, an individual b from B and a state s' such that $s' \dashv\!| \psi(b)$ and $s = \tau_{|\varphi(x)|,B} \sqcup s'$.

We should note that, on this account, the verifiers for $\forall x[\varphi(x): \psi(x)]$ and for $\forall x(\neg\varphi(x) \vee \psi(x))$ will differ, since the first will involve a relativized totality condition $\tau_{|\varphi(x)|,B}$ to the effect that the individuals of B are exactly the individuals that φ, while the second will involve an unrelativized condition τ_B to the effect that the individuals of B are exactly the individuals that there are.

Two variants on the preceding accounts should be briefly noted. First, we have taken a verifier s of $\forall x\varphi(x)$, for example, to be the *fusion* of a totality condition τ_B and the verifiers $s_1, s_2, ...$ of the instances of $\forall x\varphi(x)$. But one might think of the totality condition as a precondition for the fusion $s_1 \sqcup s_2 \sqcup ...$ of $s_1, s_2, ...$ to verify $\forall x\varphi(x)$; and this is a reason for thinking of the verifier s of $\forall x\varphi(x)$, not as the fusion of τ_B and $s_1 \sqcup s_2 \sqcup ...$, but as an ordered pair $\langle\tau_B , s_1 \sqcup s_2 \sqcup ... \rangle$, whose first component τ_B is a precondition and whose second component $s_1 \sqcup s_2 \sqcup ...$ is a post-condition or verifier proper. It turns out that this way of articulating a verifier into pre- and post-condition is very useful in a number of different contexts (and especially to presupposition).

Secondly, the verifiers of $\forall x\varphi(x)$ (or $\forall x[\varphi(x): \psi(x)]$) have been taken to involve the particular individuals $a_1, a_2, ...$ in the range of the quantifier. But it might be thought that $\forall x\varphi(x)$ is verified, in the first place, by certain general facts which, in themselves, do not involve any particular individuals. It turns out that this idea of generic verification can be developed within

[2]Allowing impossible totality states of this sort greatly simplifies the semantics and is another example of the benefits to be gained by admitting impossible states.

the framework of arbitrary objects developed in Fine [1985]. Thus the verifier of 'all men are mortal' might be taken to be the generic fact that the arbitrary man is mortal. Although I shall not go into the matter, I believe that the idea of generic verification has a wide range of interesting applications and is especially relevant to problems of 'logical omniscience'.

II. Applications

I shall consider five applications in all: two to philosophical logic - the logic of partial content and subject-matter; and three to linguistics - counterfactuals, imperatives and scalar implicature.

§1. The Logic of Partial Content

We have already noted that there appears to be an intuitive sense in which the content of one statement is part of the content of another, for which Q will generally be part of the content of $P \wedge Q$ but $P \vee Q$ will not generally be part of the content of P. In a series of publications dating from 1977, Angell developed a system of 'analytic implication' that was intended to capture the logic of this notion (Angell [1977], [1989], [2002]). It is therefore natural to wonder what is the relationship between his system and our own semantical account of the notion.

It turns out that his system exactly corresponds to our own account, at least under certain very natural assumptions. Let us take $\neg$, $\wedge$ and $\vee$, as before, to be the primitive truth-functional connectives. The notion of the content of A containing the content of B (A → B) and the notion of the content of A being equivalent to the content of B (A ↔ B) are interdefinable (with (A → B) $=_{df}$ A ∧ B ↔ A and (A ↔ B) $=_{df}$ (A → B) ∧ (B → A)); and let us, for convenience, take content equivalence (↔) rather than content containment → as primitive.

Angell's system may then be axiomatized by means of the following axioms and rules:

A1 A ↔ ¬¬A
A2 A ↔ A ∧ A
A3 A ∧ B ↔ B ∧ A
A4 (A ∧ B) ∧ C ↔ A ∧ (B ∧ C)
A5 ¬(A ∧ B) ↔ (¬A ∨ ¬B)
A6 ¬(A ∨ B) ↔ (¬A ∧ ¬B)
A7 A ∧ (B ∨ C) ↔ (A ∧ B) ∨ (A ∧ C)
R1 A(B), B ↔ C / A(C)

The sole rule is R1, which allows us to substitute provable equivalents within any give theorem.

We now adopt the inclusive semantics given under §I.6 above and insist that, in any model ***M*** = (S, ⊑, [•]), the set of verifiers and the set of falsifiers of any atomic formula should be non-empty and closed under fusion (it can be shown that this requirement will be met by all formulas whatever if it is met by the atomic formulas).

Let us say that A *analytically implies* B in a model ***M*** = (S, ⊑, [•]) if (i) every exact verifier of A (in ***M***) contains an exact verifier of B and (ii) every exact verifier of B is contained in an exact verifier of A (in conformity with our previous account of partial content). Now say that the formula A ↔ B *holds in* the model ***M*** = (S, ⊑, [•]) if A analytically implies B and B analytically implies A in ***M***. Finally, say that the formula A ↔ B is *valid* if it holds in every model. Then under the proposed semantics, we have the following completeness theorem:

A ↔ B is a theorem of Angell's system iff it is valid under the truthmaker semantics.

(Fine [2015b]; other semantics for which Angell's system is complete are given in Correia [2004] and Ferguson [2015]).

This result provides some sort of vindication both of Angell's system and of the proposed account of partial content.

§2. Subject-matter

There is an intuitive notion of subject-matter or of what a statement is about. This notion may have a different focus in different contexts. Thus it may be objectual and concern the objects talked about or it may be predicational and concern what is said about them. Our concern here will be with what one might call 'factual' focus, with what it is in the world that bears upon the statement being true or false.

A standard account of subject-matter, in more or less this sense, was developed by Lewis [1988] and subsequently elaborated by Yablo [2014], chapter 2. A subject-matter, for Lewis, is given by an equivalence relation on worlds where, intuitively, two worlds will stand in the equivalence relation when they do not differ with regard to the subject-matter. Thus if the subject-matter is the current weather in New York, then two worlds will stand in the associated equivalence relation when they do not differ with regard to the current state of the weather in New York.

Lewis has difficulty in defining *the* subject-matter of a statement. Yablo provides a more refined account of subject-matter in terms of truth-makers which removes this difficulty. But he still identifies a subject-matter with a relation on worlds, the subject-matter of a statement now being the similarity relation which holds between two worlds when they share a truthmaker.

There is, I believe, a much more satisfactory and straightforward way of defining subject-matter within the truthmaker framework statement in which no reference is made to worlds or the like (in line with our general unworldly philosophy). Suppose that the verifiers of the statement A are $s_1, s_2, \ldots$. We may then identify the subject-matter of A with the fusion $\boldsymbol{s} = s_1 \sqcup s_2 \sqcup \ldots$ of its verifiers . After all, it is these states that most directly bear upon the truth of the statement.[3]

It might be thought that this approach to subject-matter is doomed from the start. For consider the statement 'it does or does not rain' and the statement 'it does or does not snow'. Intuitively, their subject-matter is quite different, one concerning the presence or absence of rain and the other the presence or absence of snow. But on our account, the subject-matter of the first statement is the fusion of the presence and absence of rain, let us say, while the subject-matter of the second statement is the fusion of the presence and absence of snow. But these are both impossible states and therefore the same.

But this line of reasoning rests upon adopting the standard coarse-grained conception of impossible states. There is nothing in the truthmaker approach as such which requires us to adopt such a coarse-grained view. Indeed, the more natural view is one in which different

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[3] Fine [2015d]. Strictly speaking, s is the *positive* subject-matter of A. The *negative* subject-matter may be taken to be $t = t_1 \sqcup t_2 \sqcup \ldots$, where $t_1, t_2, \ldots$ are the falsifiers of A; and the *overall* subject-matter may be taken to (s, t) or $s \sqcup t$.

impossible states can be distinguished in terms of the possible states from which they have been obtained [Fine 2015a]; and, in this case, the fine grain of a given subject-matter can be recovered even though its various components have been lumped together into a single impossible state. This then is another case in which impossible states are able to earn their keep.[4]

This account of subject-matter gives rise to a simple and elegant theory of the subject. Since subject-matters are states, we can give an account of the mereology of subject-matters (subject-matter containment, overlap, disjointness etc) directly in terms of the mereology of states. We can give a simple account of the subject-matter of complex statements in terms of the subject-matters of their components. Thus where $\sigma(A)$ is the subject-matter of A, we can set:

$$\sigma(A \wedge B) = \sigma(A \vee B) = \sigma(A) \sqcup \sigma(B).$$

We can also give a simple account of the *restriction of* a given proposition to some subject-matter $\boldsymbol{s}$. For given two states s and t, let $s \sqcap t$ be their *intersection* (i.e. the fusion of all states that are a common part of s and t). We might then identify the *restriction of* the proposition $P = \{p_1, p_2, ...\}$ *to* the subject-matter $\boldsymbol{s}$ to be the proposition $\{p_1 \sqcap \boldsymbol{s}, p_2 \sqcap \boldsymbol{s}, ...\}$, obtained by restricting the verifiers of P to the given subject-matter $\boldsymbol{s}$.

Subject-matter will also play a pervasive role in the rest of the theory of truthmakers. Let me give one example from the account of partial content discussed above. When a proposition $P = \{p_1, p_2, ...\}$ is closed under fusion, then its subject-matter $\boldsymbol{p} = p_1 \sqcup p_2 \sqcup ...$ will itself be a verifier of P. This means that the second clause in the definition of partial content, viz. that every verifier of Q should be contained in a verifier of P, can be replaced by the condition that the subject matter $\boldsymbol{q}$ of Q should be part of the subject-matter $\boldsymbol{p}$ of P.

§3 Counterfactuals

Ever since the pioneering work of Stalnaker [1968] and Lewis [1973], it has been customary to to provide a semantics for counterfactuals statements in terms of possible worlds. The idea, roughly speaking, is to take the counterfactual from A to C to be true just in case the closed world - or all closest worlds, or all sufficiently close worlds - in which A is true are worlds in which C is true. If we introduce a comparative closeness relation $u \preceq_w v$ (u is as close to w as v), then we may state the third of these options, somewhat more formally, as:

$w \models A > C$ if for some world v, $v \models A$ and, for any world u, $u \models C$ whenever $u \preceq_w v$ and $u \models A$.

One familiar difficulty with this account is that it does not enable us to distinguish between the counterfactual ‘if Sue were to take the pill then she would live’ from the counterfactual ‘if Sue were to take the pill or to take the pill and the cyanide then she would live’, even though the first might well be true while the second is false. For the antecedents, ‘Sue takes the pill’ (p) and ‘Sue takes the pill or takes the pill and the cyanide’ ($p \vee (p \wedge q)$), are truth-functionally equivalent. Indeed, this difficulty is merely the tip of an iceberg. For it can be shown that the possible worlds approach (or any approach that endorses the substitution of truth-functionally equivalent antecedents) is incompatible with our intuitive judgements about certain scenarios and certain commonly accepted principles of counterfactual reasoning (Fine [2012b]).

[4] A further case is the truthmaker semantics for intuitionistic logic in Fine [2014a], which also makes use of a rich ontology of impossible states .

In any case, it is worth seeing to what extent these difficulties can be avoided under an alternative approach; and it is here that truthmaker semantics comes into its own. Instead of working with the closeness relation $u \preceq_w v$ on worlds u, v and w, we work with a transition relation $t \rightarrow_w u$ on states t and u and world w. Intuitively, $t \rightarrow_w u$ says that u is a possible outcome of imposing t on u. The clause for the truth of a counterfactual at a given world w is then given by:

$w \Vdash A > C$ if for any states t and u for which $t \Vdash A$ and $t \rightarrow_w u$, $u \,\|\!> C$.[5]

A counterfactual will be true if any exact verifier for the antecedent must transition to an inexact verifier for the consequent. So, for example, the counterfactual 'if the match were struck it would light' will be true because all of the possible outcomes of an exact verifier of 'the match is struck' will be ones which contain an exact verifier for 'the match lights'.

Let us note that this immediately takes care of the Sue example above. Indeed, each of A > C and A > C will be a consequence of A ∨ B > C (the so-called rule of Simplification), since the exact verifiers of A and the exact verifies of B are among the exact verifiers of A ∨ B. So 'if Sue were to take the pill and the cyanide then she would live' will be a consequence of 'if Sue were to take the pill or the pill and cyanide then she would live', but not of 'If Sue were to take the pill she would live'.

§4 Imperatives

There seems to be a sense in which one imperative statement may follow from others. Suppose someone says 'Turn on the light and open the door'. Then it seems that an interlocutor can legitimately say, 'So, turn on the light', thereby indicating that the one imperative follows from the other. This then raises the question: what is the logic of imperatives? When does one imperative follow from others?

The natural answer to this question is that an imperative inference will be valid just in case the corresponding indicative imperative inference is valid: the imperative Y will follow from the imperatives $X_1, X_2, \ldots, X_n$ just in case the indicative B corresponding to Y follows from the indicatives $A_1, A_2, \ldots, A_n$ corresponding to $X_1, X_2, \ldots, X_n$. Thus in the case above, 'Turn on the light' will follow from 'Turn on the light and shut the door' because 'You turn on the light' follows from 'You turn on the light and shut the door'.

However, this solution does not appear to give the right result in other cases. From You turn on the light' follows 'You turn on the light or burn the building down'. But from the imperative 'Turn on the light', it does not seem as if one can infer 'Turn on the light or burn the building down'. This is Ross' famous paradox.

How then should the semantics and logic of imperatives proceed? Again, it looks as if the framework of truthmaker semantics can provide an answer (Stelzner [1992], van Rooij

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[5] Fine [2012c]. We may also give the following more complicated clause for the exact verification of a counterfactual:

$s \Vdash A > C$ iff (i) for each t for which $t \Vdash A$ there is an s_t and a u for which $t \rightarrow s_t\, u$,
(ii) for any state t for which $t \Vdash A$ and any state u for which $t \rightarrow s_t\, u$, $u \,\|\!> C$,
(iii) s is the fusion of $\{s_t: t \Vdash A\}$;

and give a related clause for the exact falsification of a counterfactual.

[2000], Aloni & Ciardelli [2013]). Before, we took states to exactly verify or falsify an indicative statement. In the same way, we may take an action (which is a particular kind of state) to be in *exact compliance with* or *exact contravention to* an imperative statement. Thus just as your shutting the door exactly verifies the indicative statement 'You shut the door', while your shutting the door and turning on the light does not, so your shutting the door is in exact compliance with the imperative statement 'Shut the door', while your shutting the door and turning on the light is not.

We can then provide compliance and contravention conditions for logically complex imperatives that are the analogue of the verification and falsification conditions for logically complex indicatives. So, for example, we may say:

the action α is in exact compliance with the conjunctive imperative $X \wedge Y$ iff it is the fusion $\beta \sqcup \gamma$ of an action β that is in exact compliance with X and an action γ that is in exact compliance with Y.

Let the content of an imperative X be the set of actions in act compliance with the imperative. It may now be suggested that the imperative Y follows from the imperative X just in case the content of Y is part of the content of X. Thus Y will follow from X if (i) any action in compliance with X contains an action in compliance with Y and (ii) any action in compliance Y will be part of an action in complianc with X. Y must, in this sense, be a necessary means to X.

This account of imperative consequence immediately dissolves Ross' Paradox, since the content of $X \vee Y$ will not in general be part of the content of X; and it points to a surprising and intimate connection between the logic of imperatives and the logic of analytic implication.

§5 Scalar Implicature

There is a phenomenon that has been discussed in the linguistics literature under the heading 'scalar implicature'. Here is a typical example. Suppose I assert:

(1) John had toast or cereal for breakfast.

Then this is thought to have the 'implicature':

(2) John did not have both toast and cereal for breakfast.

Similarly,

(3) John took one of the candies.

is thought to have the implicature:

(4) John took at most one of the candies.

It has been supposed that, in the first of these cases, there is a sense in which (1) implies (2), since if you believe that John had both toast and cereal then it appears appropriate to register your disagreement with the words:

(5) No (well, in fact), he had both.

But it has also been supposed that the implication is not a regular semantic implication because I can consistently assert:

(6) John had toast or cereal for breakfast and perhaps he even had both.

And likewise in the second case.

How then to account for the implication when it is not a semantic implication? Many philosophers and linguists have appealed to Grice's maxims of cooperative conversation. The rough idea is that if it had been true that John had both toast and cereal for breakfast then I would have said so; and so, from my merely making the weaker claim, you can infer that he did not in fact have both.

However, the Gricean account has a hard time with such examples as:

(1)′ John had toast or cereal or toast and cereal for breakfast; and

(3)′ John took at least one of the candies

For these do not have the same implicatures - (2) and (4) - as (1) and (3); and yet the same Gricean reasoning would appear to apply.

Let me briefly suggest how these and other difficulties can be avoided under the truthmaker approach thought it will, of course, be necessary to slide over many issues.[6] We suppose that any statement is made against the background of some relevant subject-matter ***s*** (again, subject-matter comes into the picture!). Thus if I say that John had toast or cereal for breakfast, we may take the relevant subject matter to be what he had for breakfast, which we identify with the fusion of various states - his having toast, his having cereal, his having bacon and eggs, and so on.

Any subject-matter ***s*** will have an actual part $\boldsymbol{s}_{@}$, which is the fusion of all those parts of ***s*** that are actual. Where ***s*** is the relevant subject-matter of a statement A, we may now say that A is *exactly true* if $\boldsymbol{s}_{@}$ is an exact verifier of A. We might call $\boldsymbol{s}_{@}$, when ***s*** is the relevant subject-matter, the *relevant situation*. Then for A to be exactly true is for it to be exactly verified by the relevant situation.

We make the following key hypothesis:

The Origin of Scalar Implicature (OSI) Scalar implicature arises from the presupposition that a statement is not merely true but exactly true.

What we say should, in this sense, fit the facts.

Let us now see how this hypothesis can take care of the difficulties mentioned above. Consider (1). If John had both cereal and toast for breakfast, then the relevant situation would involve his having both for breakfast and so would not be an exact verifier of (1). Thus the exact truth of (1) requires that John not have both cereal and toast for breakfast. Similarly for (3). If John took more than one candy, then the relevant situation would involve his taking several candies and so would not be an exact verifier of (3) (which should be the verifier of a single instance of (3). Thus the exact truth of (3) requires that John take a single candy.

Contrast this now with (1)′ and (3)′. In this case, John having both cereal and toast would be an exact verifier of (1)' and so the exact truth of (1)′ does not require the truth of (2). Similarly for (3)′. For in this case, we may suppose that John taking several candies is an exact verifier for (3)′ (in line with the inclusive semantics for the existential quantifier in §7) and so again the implicature will be lost.

In this chapter, I have provided the merest sketch of the truthmaker approach. The abstract theory may be developed in many other directions and many other examples of its application might be given: to ground (Correia [2010], Fine [2012d], Fine [2015d]), the

[6] I first publicly presented a solution along these lines in the Jack Smart lecture of 2010; and a similar solution has been independently proposed by Robert Rooij in his [2013] and in some recent unpublished work.

determinate/determinable distinction (Fine [2011]) and the status of impossible states (Fine [2015a]) within metaphysics; to modal and deontic logic, intuitionistic logic [2014a] and the theory of logical remainder and common content (Fine [2015d], Yablo [2014], chap. 8) within philosophical logic; to quantification, anaphora, free choice disjunction, intensional descriptions (Moltmann [2015b]), cases-constructions (Moltmann [2015a}, adverbial modification (van Fraassen [1973], Hwang & Schubert[1993]), presupposition (Yablo [2014], chapter 10), the logic and semantics of questions (Ciardelli, Groenendijk, Roelefson F [2013]) and vagueness (van Rooij [2013]) within the philosophy of language and linguistics; to belief revision and closure principles for knowledge and belief (Yablo [2014], chap. 7) within epistemology; to verisimilitude (Fine [2015f], Gemes [2007], Yablo [2014],§6.7), confirmation (Gemes [1994], [1997], Yablo [2014], §6.1-5) and causal modeling within the philosophy of science; and to the psychology of reasoning (Koralus & Mascarenhas [2013]) and the frame problem within cognitive science. But I hope I have said enough to give the reader a taste of what the theory is like, of what it is capable of doing, and of how, in many respects, it is far superior to the more usual approach in terms of possible worlds.

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